

Neutrino oscillation with flavor non-eigenstates and CP-violating Majorana phases

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Abstract

We analyze neutrino oscillation for the general case when the initial neutrino is not in a pure flavor state. We show that, after such a neutrino beam propagates for a while, the probability of detecting any pure flavor state depends even on the CP-violating Majorana phases in the mixing matrix. The dependence remains even when energy spectrum of the initial beam is taken into account. We discuss various implications of this dependence.

Through solar and atmospheric neutrino data as well as ground-based oscillation experiments, we now know that neutrinos have mass, and they mix [1]. Two of the three mixing angles are already known to be non-zero, and in fact quite large, whereas for the other there exists only an upper bound. The masses themselves are not known, and cannot be known from oscillation data alone, although we have a good idea of the mass differences (more precisely, the differences of mass-squared values) between different eigenstates.

Mass matrices contain information not only about mass eigenvalues and mixing angles, but also about phases responsible for CP violation. The number of such phases depend on whether the neutrinos are Dirac fermions or Majorana fermions. For N generations of leptons, there are $\frac{1}{2}(N-1)(N-2)$ Dirac phases. In case of Majorana neutrinos, the number of phases is $\frac{1}{2}N(N-1)$, i.e., $N-1$ more phases compared to the Dirac case [2–4]. These extra CP-violating phases are sometimes called “Majorana phases”, and we will use this terminology for the sake of convenience.

An intriguing question is how to observe these phases should they exist, i.e., if the neutrinos are Majorana particles. Since Majorana neutrinos are obtained in theories where lepton number is violated, the observability of the Majorana phases

was discussed in the context of lepton number violating processes [4, 5], and for a long time it was believed that these phases can be observed only in such processes. Later, it was shown [6, 7] that lepton number violation in the process is not necessary in obtaining information about the Majorana phases.

Inspired by this knowledge, we can ask the question whether the Majorana phases can be observed in neutrino oscillation experiments. This is what we try to do in this article, for neutrinos oscillating in the vacuum and also in matter.

It is best to address this question assuming there are just two generations of neutrinos, where the analysis can be carried out analytically. In this case, the mixing matrix contains only one phase and it is of the Majorana type. The flavor states ν_e and ν_μ are related to the mass eigenstates through the relation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & e^{i\alpha} \sin \theta \\ -e^{-i\alpha} \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \quad (1)$$

Suppose now we start with a monochromatic neutrino beam, with energy E , which is not necessarily in one of the flavored states, but is a general superposition of the two flavors:

$$|\psi(0)\rangle = A|\nu_e\rangle + B|\nu_\mu\rangle \quad (2)$$

$$= (A \cos \theta - B e^{-i\alpha} \sin \theta) |\nu_1\rangle + (A e^{i\alpha} \sin \theta + B \cos \theta) |\nu_2\rangle, \quad (3)$$

where A and B are real, and $A^2 + B^2 = 1$. We could have been more general and assumed that there is a relative phase between A and B , but that is not necessary for the argument that we are going to present. Time evolution of this state gives, apart from an overall phase that is unimportant, the result

$$|\psi(t)\rangle = (A \cos \theta - B e^{-i\alpha} \sin \theta) |\nu_1\rangle + (A e^{i\alpha} \sin \theta + B \cos \theta) e^{-2i\delta} |\nu_2\rangle, \quad (4)$$

where

$$\delta = \frac{m_2^2 - m_1^2}{4E} t, \quad (5)$$

m_1 and m_2 being the mass eigenvalues of ν_1 and ν_2 .

We can now ask the question what is the probability of finding one of the flavors, say ν_e , in this resulting beam. The answer is given by

$$P_{\nu_e}(t) \equiv |\langle \nu_e | \psi(t) \rangle|^2. \quad (6)$$

Using Eq. (1) to express ν_e in terms of the eigenstates and performing a few steps of trivial algebra, we obtain

$$\begin{aligned} P_{\nu_e}(t) = & A^2(1 - \sin^2 2\theta \sin^2 \delta) + B^2 \sin^2 2\theta \sin^2 \delta \\ & - 2AB \sin 2\theta \sin \delta (\cos^2 \theta \sin(\alpha + \delta) + \sin^2 \theta \sin(\alpha - \delta)). \end{aligned} \quad (7)$$

Clearly, if $A = 1$, this will denote the survival probability of ν_e 's in a beam that started as ν_e , and in this case our expression reduces to the familiar expression for

that case. Our result also agrees with the analysis of neutrino oscillation probabilities performed [8] for A and B both non-zero but assuming Dirac neutrinos, which implied $\alpha = 0$. The same analysis can hold for Majorana neutrinos if CP violation is neglected. But our expression is more general than all these results, and it shows that if neither A nor B vanishes, i.e., if the initial beam is not in a pure flavor state, then the oscillation probability depends on the Majorana phase α .

What is more important, the dependence on α does not get washed out even if we have a neutrino beam with a large spread of energy which travels through a long enough duration of time. In this case, we need to take averages of the δ -dependent quantities over the energy spectrum. Quantities like $\sin \delta$ or $\cos \delta$ vanish under such averaging, but $\sin^2 \delta$ averages to $\frac{1}{2}$, and we obtain

$$P_{\nu_e}(t) = A^2(1 - \frac{1}{2} \sin^2 2\theta) + \frac{1}{2} B^2 \sin^2 2\theta - AB \sin 2\theta \cos 2\theta \cos \alpha. \quad (8)$$

We now proceed to derive the corresponding formulas for oscillations in matter. We denote the 2×2 Hamiltonian in the flavor basis by

$$\mathbb{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}. \quad (9)$$

The Hamiltonian has to be hermitian, irrespective of whether we have Dirac type of neutrinos or Majorana type, and so we must have H_{11} and H_{22} real and $H_{21} = H_{12}^*$. Now, if there is a matrix \mathbb{U} of the form shown in Eq. (1) that makes $\mathbb{U}^\dagger \mathbb{H} \mathbb{U}$ diagonal, the parameters of this matrix are given, in terms of the elements of \mathbb{H} , by

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2 |H_{12}|}{H_{22} - H_{11}} \right), \quad \alpha = \arg(H_{12}). \quad (10)$$

For neutrinos passing through matter, the Hamiltonian depends on the density [9]. Density corrections appear in the diagonal elements of the Hamiltonian written in the flavor basis. As a result, α does not change with density, although θ does.

So now, suppose the initial state produced is the superposition of ν_e and ν_μ , as given in Eq. (2). At the production point, the density is such that the effective mixing angle is θ_0 . Thus,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & e^{i\alpha} \sin \theta_0 \\ -e^{-i\alpha} \sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_0, \quad (11)$$

where the subscript “0” indicates the production point in matter. Thus,

$$|\psi(0)\rangle = (A \cos \theta_0 - B e^{-i\alpha} \sin \theta_0) |\nu_1\rangle_0 + (A e^{i\alpha} \sin \theta_0 + B \cos \theta_0) |\nu_2\rangle_0. \quad (12)$$

The probability that this state is in the first eigenstate is given by $|A \cos \theta_0 - B \sin \theta_0 e^{-i\alpha}|^2$. In the adiabatic case, this probability remains constant, although the eigenstate, as a superposition of the flavor states, changes with density. If at the end of the journey the neutrino is detected in the vacuum where Eq. (1) holds,

the probability of finding a $|\nu_e\rangle$ in this state is just $\cos^2 \theta$. Similarly adding the contribution from the other eigenstate, we obtain

$$\begin{aligned} P_{\nu_e}^{(\text{ad})}(t) &= |A \cos \theta_0 - B \sin \theta_0 e^{-i\alpha}|^2 \cos^2 \theta + |A \sin \theta_0 e^{i\alpha} + B \cos \theta_0|^2 \sin^2 \theta \\ &= \frac{1}{2} A^2 (1 + \cos 2\theta_0 \cos 2\theta) + \frac{1}{2} B^2 (1 - \cos 2\theta_0 \cos 2\theta) \\ &\quad - AB \sin 2\theta_0 \cos 2\theta \cos \alpha. \end{aligned} \quad (13)$$

The superscript ‘ad’ reminds us that this is the result if the adiabatic condition, described above, holds. This is, of course, the energy-averaged expression, because we have neglected the oscillatory terms by working with an explanation in terms of probabilities [10] and not of amplitudes. But even in this expression, there is an α -dependence.

If the conditions are non-adiabatic, then neutrinos can jump from one eigenstate to another. If this jump probability is called P_{jump} , then the probability of finding ν_e would be given by [11, 12]

$$P_{\nu_e} = (1 - P_{\text{jump}}) P_{\nu_e}^{(\text{ad})} + P_{\text{jump}} (1 - P_{\nu_e}^{(\text{ad})}). \quad (14)$$

The jumping probability depends on the expression of the eigenvalues in terms of the parameters that appear in the Hamiltonian in the flavor basis. These eigenvalues are independent of α and therefore the jumping probability is independent of α as well. However, the expression for P_{ν_e} still depends on α through the adiabatic probability that appears in this expression.

There are claims in the literature that the Majorana phases are unobservable in neutrino oscillation experiments, whether in vacuum [2–4] or in matter [13]. However, these claims have all been made in the context of initial neutrino states of pure flavor. This is consistent with our results since the α -dependence in all relevant formulas appear with a co-efficient AB , and therefore vanish for pure flavor states which have either $A = 0$ or $B = 0$.

Presence of a non-zero value of the CP-violating phase α can introduce some qualitative features in neutrino oscillation probabilities. As an example, suppose we have an initial neutrino beam with some known admixture of two flavors. If $\alpha = 0$, the probability of finding ν_e after this beam travels for a time t in the vacuum is given by

$$P_{\nu_e}(t) \Big|_{\alpha=0} = A^2 - \left[(A^2 - B^2) \sin^2 2\theta + 2AB \sin 2\theta \cos 2\theta \right] \sin^2 \delta. \quad (15)$$

If the initial state was such that both A and B are positive with $A > B$, the right hand side of this expression is always less than A^2 , assuming $\theta < \pi/4$ which is certainly valid for ν_e - ν_μ oscillation, as inferred from various solar and terrestrial experiments [1]. However, if $\alpha \neq 0$, we need to use the expression given in Eq. (7), and $P_{\nu_e}(t)$ is no longer guaranteed to be less than A^2 . Thus, if one observes a reinforcement of the dominant component of the beam in an oscillation experiment, it definitely signals a non-zero value of the phase α .

Let us conclude this article with a qualitative feeling for the magnitude of the importance of α oscillation formulas, which might help in devising experimental

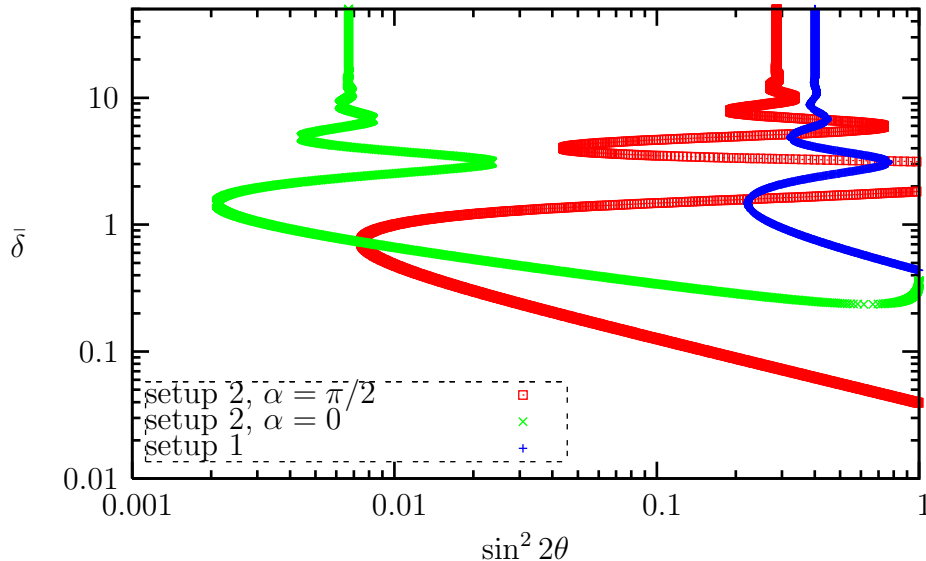


Figure 1: Various equal probability contours. Details have been given in the text.

techniques for finding α from neutrino oscillation experiments. We will do this by considering fictitious data coming from two different oscillation experiments. It has been argued [14] that the analysis of data from a single oscillation experiment is conveniently done by introducing the dimensionless parameter

$$\bar{\delta} \equiv \frac{m_2^2 - m_1^2}{4\langle E \rangle} t, \quad (16)$$

where $\langle E \rangle$ is the average energy in the incoming beam. Different experiments can have different average energy in the incoming beam and different distances between the source and the detector, and consequently different values of $\bar{\delta}$ even when they are measuring oscillation between the same two flavors. In order to provide an illustrative example of the point we are discussing here, we consider, for the sake of simplicity, that two experiments have the same $\langle E \rangle$ and the same source-detector distance (or at least the same ratio of the two quantities just mentioned), and therefore the same value of $\bar{\delta}$. One of these experiments is working with an initial beam that is purely ν_e , and observes that $P_{\nu_e} = 0.8$ at the detection point. We can ask which values of $\bar{\delta}$ and the mixing angle can make it possible. The answer will be independent of α since the initial beam was purely ν_e , and has been shown in Fig. 1. The shape of this line of course depends on the energy spectrum of the initial beam. For the sake of definiteness, we have taken a Gaussian energy spectrum with a standard deviation equal to $0.2\langle E \rangle$.

Now suppose that in the second experiment, the initial neutrino beam has $A = 0.8$ and $B = 0.6$ in the initial beam, and at the detector, one finds $P_{\nu_e} = 0.6$. The energy spectrum is the same. If $\alpha = 0$, the equal probability contour is given by the line that reaches leftmost in Fig. 1. It is worth pointing out a special feature of this line compared to the line obtained from the first experiment. Near the bottom end of the line, we have two solutions of the mixing angle for the same value of $\bar{\delta}$.

This happens because, when A and B are both non-zero, the θ -dependence of the expression in Eq. (7) is not monotonic.

Anyway, this line is clearly inconsistent with the result of the first experiment. However, if $\alpha \neq 0$, this need not be the case. We show the contour for $\alpha = \pi/2$ in Fig. 1 with a thick solid line, which has clearly some intersections with the curve from the first experiment. Thus, non-zero values of α will make the results of the two experiments consistent with each other, and the value of α can even be determined from such data from two experiments.

Real experiments, of course, will have some error bar on the detected probability, so there will be a band rather than a contour corresponding to a single experiment. There will be other complications in the analysis because the energy spectra of no two experiments will be the same. Obtaining an initial beam which is not a flavor eigenstate is not straight forward as well. Here, we have only discussed the matters of principle and indicated possible ways in which the CP-violating phase α might be detected from neutrino oscillation data.

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